## Linear Transformations Worksheet

Opening Question: You are a graphic designer developing a new font. Your current task is to italicize the letter ' $N$ '. If you draw ' $N$ ' in $\mathbb{R}^{2}$, can you create a function to italicize it? Is your function a linear transformation? Will it work for other letters?

1. You draw the letter ' N ' in $\mathbb{R}^{2}$ and then transform it into italics. Viewing each point on the ' N ' as a vector in $\mathbb{R}^{2}$, can you find a linear transformation that moves the vectors to the italicized positions?


2. Next, you want to take a mirror image of your characters (see below). Using the same approach as before, can you find a linear transformation that does this?


3. Reflect back on $\# 1$ and $\# 2$ and answer the following questions:
(a) How do you build the standard matrix of a linear transformation?
(b) What is the least amount of information we need about what $T$ does to vectors in $\mathbb{R}^{n}$ before we know what $T$ does to all vectors in $\mathbb{R}^{n}$ ?
4. Now, consider the following transformations of the letter ' N '. Which are not possible to achieve by a linear transformation? Change the image as little as possible so that it is the result of a linear transformation.

5. Go further:
(a) What happens when you italicize then mirror? What about doing it in the other order?
(b) If you had a linear transformation from $\mathbb{R}^{3} \rightarrow \mathbb{R}^{5}$, what is the minimum amount of information you need to build the standard matrix for the transformation?
(c) What must a linear transformation do to a line?
